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Mathematical Modeling of Blood Flow in a Stenosed Artery: A Two-Layered Approach with Peripheral Layer Effects

Dr. Goutam Das
Assistant Professor, Department of Mathematics, Madhyamgram B.Ed. College, Madhyamgram (W.B.), India
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Abstract

This study investigates a two-layered mathematical model of blood flow in a stenosed artery, which includes a cosine-shaped constriction to represent the arterial narrowing and a peripheral plasma layer. The model differentiates between the central core region, which contains cellular components, and the peripheral plasma layer, which is devoid of red blood cells. A mathematical derivation of the interface geometry between these two layers captures the intricate interaction brought on by stenosis. In order to get insight into how the peripheral layer affects the dynamics of blood flow overall, numerical analysis is used to assess important hemodynamic parameters as velocity profiles, wall shear stress, and flow resistance. The study highlights the advantages of the proposed model by comparing its results with those of existing models, showcasing its improved accuracy in representing physiological conditions. This work contributes to a deeper understanding of blood flow mechanics in stenosed arteries and offers potential applications in the diagnosis and treatment of cardiovascular diseases.

Keywords: *Blood Flow, Stenosed Artery, Two-layered Model, Peripheral Plasma Layer, Cosine-shaped Constriction, Hemodynamic Parameters, Velocity Profiles, Wall Shear Stress, Flow Resistance, Cardiovascular Diseases*

Introduction

The study of blood flow dynamics in stenosed arteries has attracted a lot of interest because of its important implications for comprehending cardiovascular disorders. The narrowing of blood vessels that characterizes stenosis interferes with regular blood flow and frequently results in life-threatening illnesses including heart attacks and strokes. In order to understand the intricacies of these physiological processes and provide information that can guide diagnostic and treatment plans, mathematical modeling is an essential tool.

This research work investigates the mathematical modeling of blood flow in a stenosed artery using a two-layered technique that takes into consideration peripheral layer effects. The two-layered model makes a distinction between the peripheral plasma layer, where cell-free fluid primarily flows, and the core region, which is dominated by the presence of erythrocytes. This distinction is essential for describing the non-Newtonian characteristics and shear-thinning behavior of blood, especially in narrow and irregularly shaped artery sections.

The inclusion of peripheral layer effects in the model allows for a more thorough examination of flow features such pressure drop, wall shear stress, and velocity profiles. These metrics are essential for comprehending the clinical and physiological effects of vascular stenosis. The suggested approach seeks to improve the clinical relevance and prognostic accuracy of blood flow simulations in stenosed arteries by bridging the gap between theoretical concepts and physiological realities.

Review of Related Literature

The mathematical modeling of blood flow in stenosed arteries has been extensively studied over the past decades, with significant advancements in understanding the complexities of arterial flow dynamics. Early studies emphasized the non-Newtonian nature of blood and the critical role of arterial geometry in influencing hemodynamic parameters. Ikbal et al. (2009) conducted an unsteady analysis of a two-layered blood flow through a flexible artery under stenotic conditions. They represented the flowing blood with a two-fluid model, which included a core region of suspension containing all erythrocytes, believed to be Eringen's micropolar fluid, and a plasma layer devoid of any cells, acting as a Newtonian fluid. A mathematical model for analyzing two-dimensional pulsatile blood flow through restricted arteries while subjected to body acceleration and a magnetic field has been developed by Haghighi and Aliashraf (2018). To better replicate in-vivo conditions, their study treated the artery as an elastic cylindrical tube, assuming that the constriction's geometry was time-dependent. The blood flow was thought to be fully developed, nonlinear, and incompressible, reflecting the complexity of the body. They used the Crank-Nicolson numerical technique with suitable beginning and boundary conditions to solve the nonlinear momentum and continuity equations. Their research advances our knowledge of the dynamics of blood flow in diseased arteries, especially when subjected to outside factors like fluctuating acceleration and magnetic fields. Chitra and Karthikeyan (2017) presented a mathematical model that includes changeable slip at the arterial wall and axially variable peripheral layer thickness to study blood flow through a stenosed artery. The core and periphery of the model were treated as non-Newtonian fluids with different changing viscosities, taking into account a two-layered structure. They gained important knowledge about the dynamics of flow in stenosed arteries by using the Frobenius method to solve the governing equations for laminar, incompressible, and fully developed non-Newtonian flow under slip boundary conditions. Biswas and Chakraborty (2010) investigated a two-fluid model to study pulsatile blood flow through an artery with mild stenosis. They used a Bingham plastic fluid model for the core region, which contained suspended erythrocytes, and a Newtonian fluid model for the periphery plasma layer. The study provided insights into the behavior of blood flow in stenosed arteries by examining the impact of body acceleration, the non-Newtonian nature of blood, and velocity slip at the arterial wall. A mathematical model of blood flow in a stenosed artery with a cosine-shaped constriction that included a peripheral plasma layer was studied by Kumar et al. (2022) using two layers. The model depicted the blood flow as consisting of a core region and a peripheral plasma layer free of red blood cells. In order to gain insight into how the peripheral layer affects blood flow dynamics in stenosed arteries, the shape of the interface between these two regions was established and the outcomes were numerically assessed and analyzed.

Therefore, the goal of this work is to use a two-fluid model to study blood flow in narrow blood vessels. The mathematical model examines a two-layered model of blood, with a peripheral layer of plasma (Newtonian fluid) and a core region of suspension of all the erythrocytes (small, spherical, non-flexible particles), which is assumed to be a particle-fluid suspension (i.e., a suspension of red cells in plasma).

Mathematical Analysis

Consider the blood flow in a cylindrical tube with axisymmetric moderate stenosis using a two-layer model. Two fluid models, plasma and red blood cells, represent blood in this concept. Thus, the shape of stenosis is considered to be symmetrical, and it is given by:

$$\frac{(R,R_1)}{R_0} = \begin{cases} (1,\alpha) - (\delta_s,\delta_i)\left[1+\cos\left(\frac{2\pi}{L_0}\right)(z-d-L_0)\right], & d \le z \le L_0 \\ (1,\alpha), & \text{otherwise} \end{cases} \tag{1}$$

such that $(\delta_s,\delta_i) \le R_0$ at $z = d + L_0$, $\frac{2R_0}{L_0} \approx O(1)$ and $2\delta_s, R_0 \le z \le L_0$.

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where L_0 is the stenosis length, d is the stenosis location, α is the ratio of the central core radius to tube in the unobstructed region, (δ_s, δ_i) is the maximum height of stenosis, and $R(z)$ is the radius of the tube with stenosis and R_0 is the constant radius.

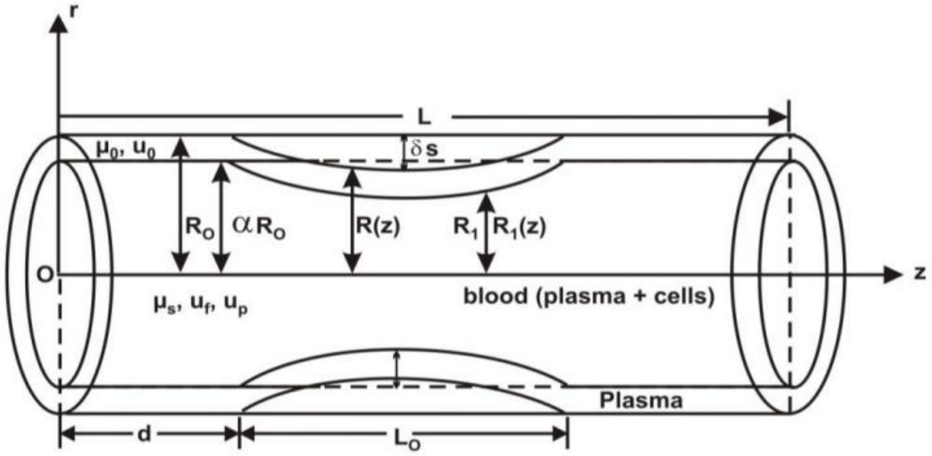

Figure 1: Arterial stenosis geometry with a peripheral layer

The Navier-Stokes equation of motion that governs the model is as follows:

$$-\frac{dp}{dz} + \mu_s\left[\frac{\partial^2 u_f}{\partial r^2} + \frac{1}{r}\frac{\partial u_f}{\partial r}\right] + KS(u_p - u_f) = 0, \text{ at } 0 \le r \le R_1 \quad (2)$$

$$-\frac{dp}{dz} + \mu_0\left[\frac{\partial^2 u_0}{\partial r^2} + \frac{1}{r}\frac{\partial u_0}{\partial r}\right] = 0, \quad \text{at } R_1 \le r \le R_0 \quad (3)$$

where $K = \frac{H}{1-H}$, axial velocity is represented by u_f and u_p for the fluid and plasma, respectively; viscosity and axial velocity of the fluid in the peripheral region are represented by μ_s and u_0; drag coefficient of interaction between the two layers (phase) is represented by S; and the constant volume fraction density of the particles, or hematocrit, is represented by H.

The empirical correlation between the viscosity of suspension (μ_s) and the drag coefficient of interaction can be expressed as follows:

$$S = \frac{9\mu_0}{2\alpha_0^2}\frac{\left[4+3\sqrt{(8H-3H^2)}+3H\right]}{(2-3H)^2}, \quad (4)$$

and $$\mu_s = \frac{\mu_0}{1-qH}, \quad (5)$$

where $q = 0.070\exp[2.49H + (1107^0 K/T)\exp(-1.69H)]$, α_0 is the radius of the particle, and its absolute temperature is T.

The following are the boundary conditions:

$$u_0 = 0 \quad \text{at } r = R \quad (6)$$

$$\begin{cases} u_0 = u_f \\ \tau_0 = \tau_f \end{cases} \quad \text{at } r = R_1 \quad (7)$$

$$\frac{\partial u_f}{\partial r} = \frac{\partial u_p}{\partial r} = 0 \quad \text{at } r = 0 \quad (8)$$

and $$\frac{\partial p}{\partial z} = 0 \quad \text{at } r = R_1 \quad (9)$$

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where the shearing stresses in the core and peripheral regions are denoted by τ_0 and τ_f, respectively. The velocities u_0, u_f, and u_p may now be derived from the expressions (2) and (3) using the boundary conditions (6)-(8) as follows:

$$u_0 = -\frac{R^2}{4\mu_0}\frac{dp}{dz}\left\{1-\left(\frac{r}{R}\right)^2\right\}, \quad \text{at } R_1 \le r \le R \quad (10)$$

$$u_f = -\frac{R^2}{4(1-H)\mu_0}\left[\frac{dp}{dz}\left\{\mu\left(\frac{R_1}{R}\right)^2-\left(\frac{r}{R}\right)^2\right\}+(1-H)\left\{1-\left(\frac{R_1}{R}\right)^2\right\}\right], \quad \text{at } 0 \le r \le R_1 \quad (11)$$

$$u_p = -\frac{R^2}{4(1-H)\mu_0}\left[\frac{dp}{dz}\left\{\mu\left(\frac{R_1}{R}\right)^2-\left(\frac{r}{R}\right)^2\right\}+(1-H)\left\{1-\left(\frac{R_1}{R}\right)^2+\frac{4(1-H)\mu_0}{SR^2}\right\}\right], \quad \text{at } 0 \le r \le R_1 \quad (12)$$

The flux rate (Q) can be computed as follows:

$$Q = Q_0 + Q_f + Q_p$$

$$Q = 2\pi\left[\int_{R_1(z)}^{R(z)} r u_0 dr + (1-H)\int_0^{R_1(z)} r u_f dr + H\int_0^{R_1(z)} r u_p dr\right]$$

$$= -\frac{\pi R_0^4}{8(1-H)\mu_0}\frac{dp}{dz}\left[(1-H)\left\{\left(\frac{R}{R_0}\right)^4-\left(\frac{R_1}{R_0}\right)^4\right\}+\mu\left(\frac{R}{R_0}\right)^4+\eta^2\left(\frac{R_1}{R_0}\right)^4\right] \quad (13)$$

where $\mu = \frac{\mu_0}{\mu_s}$ and $\eta^2 = \frac{8H(1-H)\mu_0}{SR^2}$ represents the non-dimensional suspension parameter.

In this case, we are applying the ideas that the overall flux is equal to the sum of the fluxes in the core and the periphery. The relationships are obtained as follows:

$R_1 = \alpha R$ and $\delta_i = \delta_s$

The aforementioned expression can then be simplified to:

$$\frac{dp}{dz} = -\frac{8(1-H)\mu_0 Q}{\pi R_0^4}\frac{1}{\left(\frac{R}{R_0}\right)^2\left[\beta\left(\frac{R}{R_0}\right)^2+\alpha^2\eta^2\right]} \quad (14)$$

where α is a constant quantity, $\beta = (1-H)(1-\alpha^4)\mu'\alpha^4$, and the wall shear stress is:

$$\bar{\lambda} = (1-H)\left[\frac{1-\frac{L_0}{L}}{\beta+\alpha^2\eta^2}+\int_0^{2\pi}\frac{d\phi}{(a+b\cos\phi)^2\{\beta(a+b\cos\phi)^2+\alpha^2\eta^2\}}\right] \quad (15)$$

where $\phi = \pi-\left(\frac{2\pi}{L_0}\right)\left(z-d-\frac{L_0}{2}\right)$, $a = 1-\frac{\delta_s}{2R_0}$ and $b = \frac{\delta_s}{2R_0}$.

We have now determined the wall shear stress:

$$\overline{\tau_R} = \frac{(1-H)}{\left(\frac{R}{R_0}\right)\left[\beta\left(\frac{R}{R_0}\right)^2+\alpha^2\eta^2\right]} \quad (16)$$

as well as the shearing stress on the wall at the highest point of the stenosis as follows:

$$\overline{\tau_s} = \frac{(1-H)}{\left(1-\frac{\delta_s}{R_0}\right)\left[\beta\left(1-\frac{\delta_s}{R_0}\right)^2+\alpha^2\eta^2\right]} \quad (17)$$

such that; $\bar{\lambda} = \frac{\lambda}{\lambda_H}$, $\bar{\tau} = \frac{\tau_R}{\tau_H}$

where $\lambda = \frac{1}{Q}\int_0^L\left(-\frac{dp}{dz}\right)dz$, $\lambda_H = \frac{8\mu_0 L_0}{R_0^4}$, $\tau_H = \frac{4\mu_0 Q}{R_0^3}$.

The resistance to flow and the wall shear stress for a normal artery without the particle phase are denoted by λ_H and τ_H respectively.

Numerical Results and Discussion

Blood flow in blood vessels with small diameters has been described using a two-layer model that consists of a core region of plasma-suspended erythrocytes and a peripheral layer of plasma (Newtonian fluid). The constant values that we are taking into consideration for the results are as follows:

$R_1 = 1, \alpha = 0.920, , R = 0.5, , H = 0.2, 0.1, , \frac{\delta_s}{R_0} = 0.05, 0.10, \frac{L_0}{L} = 1.0, 0.5,$

$\mu_0 = 1.24, \mu_s = 4.03, \mu' = 0.3077, \varepsilon = 2.98, \tau_s = 1.433, \lambda_H = 1.313.$

This study examines how red blood cells and the peripheral layer affect blood flow characteristics when an artery has minimal stenosis. The axial velocity profiles (u_f, u_p, and u_0) were calculated using the theory that was presented (equations (10)–(12)), the corresponding model that Haynes(1960) derived (i.e., using erythrocytes-plasma suspension to represent blood in the core region similar to the present proposed model), and the steady flow model of Bugliarello and Sevilla(1970).

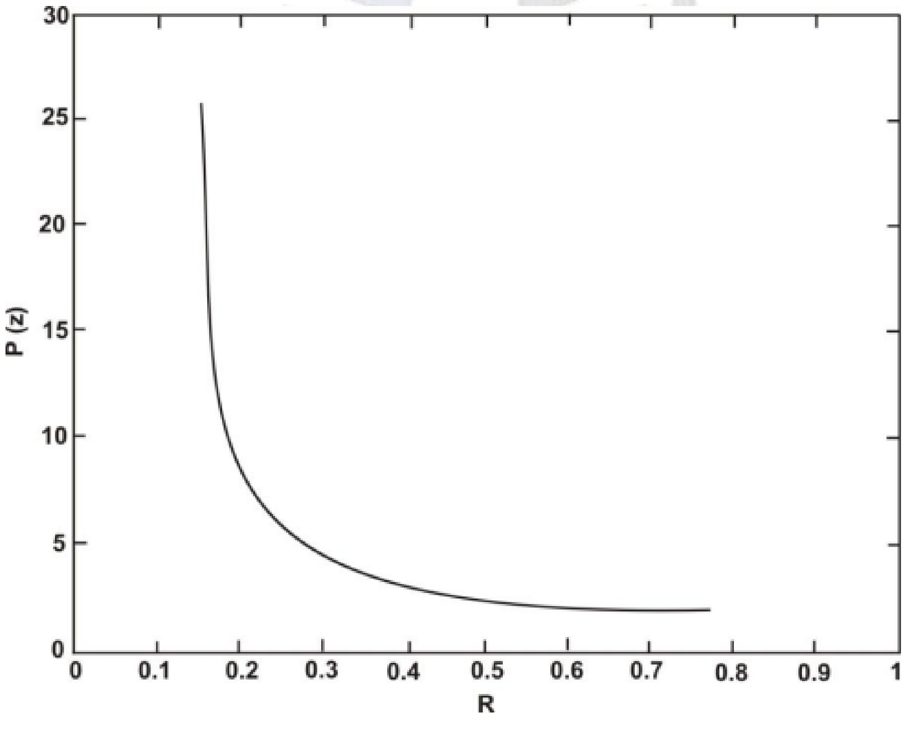

Figure 2: Plotting P (z) versus Radius for H = 0.1

ln order to demonstrate the numerical significance of the mathematical model presented in this study and to explore its outcomes statistically. to assess the analytical findings for flow rates and velocity profiles derived from equations (10)-(15) for a range of parameter values. The numerical findings achieved in this model when compared to a two-layered fluid model with a peripheral layer present are shown in Figure 2.

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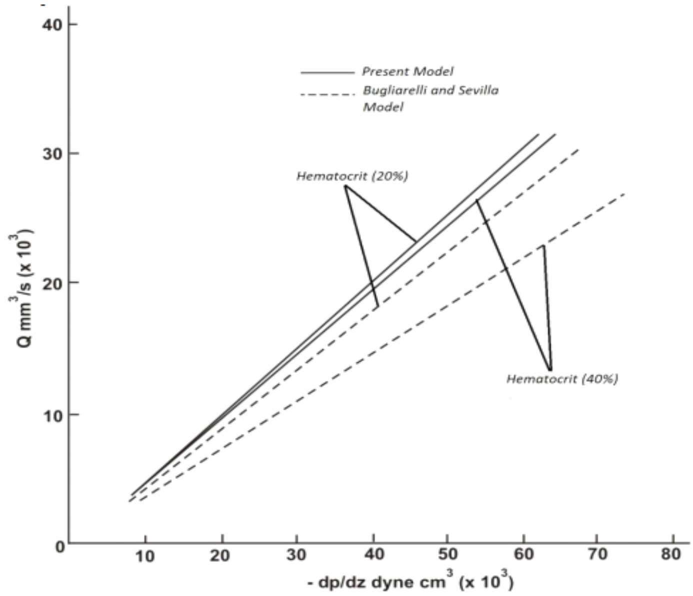

Figure 3: Relationship between pressure and flow rate in a vessel for various hematocrit.

Figure 3 shows the experimentally tested volumetric flow rate Q at various hematocrit percentages plotted against the pressure gradient $\left(-\frac{dp}{dz}\right)$ suggested in the model by equation (13). It should be mentioned that the flow rate Q magnitude measured in Bugliarello and Sevilla(1970) was especially low for a pressure gradient.

Conclusion

The impact of peripheral layer and red cell content on blood flow parameters in the presence of minor artery stenosis is examined in this study. According to the findings, a two-core blood flow model's flow characteristics are larger than those of a single-layered model. On the other hand, compared to a one-layered model, the magnitudes are smaller in a two-layered blood flow model. The results highlight the growing importance of the peripheral layer, especially in particle-free flow, as blood channel diameter reduces. This suggests that the peripheral layer, particularly in narrower arteries, is essential for controlling blood flow.

The findings further highlight the relationship between vessel size and flow parameters. Its physiological significance is further supported by the fact that a significant increase in flow may be expected as a result of its existence. The study effectively establishes an indirect association between red cell content and flow parameters, although acknowledging certain limitations. This relationship offers insightful information and is a step up from conventional single-phase Newtonian models that have been frequently studied in the past.

The significance of the two-layered approach is further supported by the comparative analysis that is provided (as shown in Figure 3). In particular, the study indicates that arteries with diameters $\leq 200\ \mu m$, where the peripheral layer and red cell concentration have a significant influence, are better suited for the two-layered blood flow model. In the meantime, in situations where red cell effect predominates, the one-layered model is still relevant for vessels with similar sizes. All things considered, this study advances our knowledge of hemodynamics in stenosed arteries and underscores how multi-layered models may be used to better represent the intricacies of blood flow.

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